INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
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MARKSCHEME

November 1999

MATHEMATICS

Higher Level

Paper 1

13 pages

Paper 1 Markscheme

Instructions to Examiners

1 Method of Marking

(a) All marking must be done using a **red** pen.

(b) In this paper, the maximum mark is awarded for a **correct answer**, irrespective of the method used. Thus, if the correct answer appears in the answer box, award the maximum mark and move onto the next question; in this case there is no need to check the method.

(c) If an **answer is wrong**, then marks should be awarded for the method according to the markscheme. (A correct answer incorrectly transferred to the answer box is awarded the maximum mark.)

2 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

C Marks awarded for **Correct** answers (irrespective of working shown)

R Marks awarded for clear **Reasoning**

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

(i) penalise the error when it **first occurs**;

(ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;

(iii) award **M** marks for a correct method and **A(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The errors made by a candidate may be: arithmetical errors; errors in algebraic manipulation; errors in geometrical representation; use of an incorrect formula; errors in conceptual understanding.

The following illustrates a use of the **follow through** procedure.

Markscheme		Candidate's Script	Marking	
\$ 600 × 1.02	**M1**	Amount earned = \$ 600 × 1.02	✓	**M1**
= \$ 612	**A1**	= \$602	×	**A0**
\$ (306 × 1.02) + (306 × 1.04)	**M1**	Amount = 301 × 1.02 + 301 × 1.04	✓	**M1**
= \$ 630.36	**A1**	= \$ 620.06	✓	**A1(ft)**

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

(i) **fewer** marks should be awarded at the discretion of the Examiner;

(ii) marks awarded should be followed by '**(d)**' (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

(i) a mark should be awarded followed by '**(d)**' (to indicate that the marks have been awarded at the **discretion** of the Examiner);

(ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

(b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

(c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , 1·7 , 1,7 ; different forms of vector notation such as $\vec{u}$, $\bar{u}$, $\underset{\sim}{u}$; $\tan^{-1} x$ for arctan x.

5 Accuracy of Answers

(a) In the case when the accuracy of the answer is **specified in the question** (for example: "find the size of angle A to the nearest degree") the maximum mark is awarded **only if** the correct answer is given to the accuracy required.

(b) When the accuracy is **not** specified in the question, then the general rule applies:

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

In this case, the candidate is **penalised once only IN THE PAPER** for giving a correct answer to the wrong degree of accuracy. Hence, on the **first** occasion in the paper when a correct answer is given to the wrong degree of accuracy maximum marks are **not** awarded, but on **all subsequent occasions** when correct answers are given to the wrong degree of accuracy then maximum marks **are** awarded.

NOVEMBER 1999

Additional instructions for Assistant Examiners

1. SAMPLES

All examiners are reminded that samples should be sent to the Team Leader by the fastest means possible. IBCA will reimburse examiners for any costs incurred.

2. PAPER 2 EXAMINERS – PART MARKS

Assistant examiners are asked to indicate on candidates' scripts the part marks that they have allocated for each part. To help identify these marks, they have now been incorporated into the markschemes. Where the markscheme has part marks *e.g. [3 marks]* after a part solution, assistant examiners should note the candidates' marks for that part of the question alongside the solution. They should also write down the total for the question at the end of each question.

1. (a) $a_1 = 5$ and $d = 8$ *(M1)*

$a_n = a_1 + (n-1)d$

$a_n = 8n - 3$ *(A1)*

(b) $8n - 3 < 400$ *(M1)*

$8n < 403$

$n < 50.375$ or $n < 50\frac{3}{8}$ or $n < 51$

Therefore, there are 50 terms less than 400. *(A1)*

Answers : (a) $a_n = 8n - 3$ *(C2)*

(b) 50 *(C2)*

2. $\det(A - kI) = 0$

$\Rightarrow \begin{vmatrix} 3-k & 2 \\ -1 & -k \end{vmatrix} = 0$ *(M1)*

$\Rightarrow k^2 - 3k + 2 = 0$ *(M1)*

$\Rightarrow (k-2)(k-1) = 0$

$\Rightarrow k = 1, 2$ *(A2)*

Answers : $k = 1$, or $k = 2$ *(C4)*

3. (a) An equation of the plane is
$2x - y + 3z = 9$. *(M1)(A1)*

OR $r = \begin{pmatrix}2\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\0\end{pmatrix} + \mu\begin{pmatrix}0\\3\\1\end{pmatrix}$ *(M1)(A1)*

OR $r \cdot \begin{pmatrix}2\\-1\\3\end{pmatrix} = \begin{pmatrix}2\\1\\2\end{pmatrix} \cdot \begin{pmatrix}2\\-1\\3\end{pmatrix} = 9$ *(M1)(A1)*

(b) $(a, a-1, a-2)$ lies on the plane if $2a - (a-1) + 3(a-2) = 9$ *(M1)*
This gives $a = \frac{7}{2}$. *(A1)*

OR $(a, a-1, a-2)$ lies on the plane if $a = 2 + \lambda, a - 1 = 1 + 2\lambda + 3\mu$ and $a - 2 = 2 + \mu$.
Thus $a - 1 = 1 + 2(a-2) + 3(a-4)$ *(M1)*
$\Rightarrow a = 3\frac{1}{2}$ or $\frac{7}{2}$ *(A1)*

OR $(a, a-1, a-2)$ lies on the plane if $\begin{pmatrix}a\\a-1\\a-2\end{pmatrix} \cdot \begin{pmatrix}2\\-1\\3\end{pmatrix} = 9$ *(M1)*
$\Rightarrow 2a - (a-1) + 3(a-2) = 9$
$\Rightarrow a = 3\frac{1}{2}$ *(A1)*

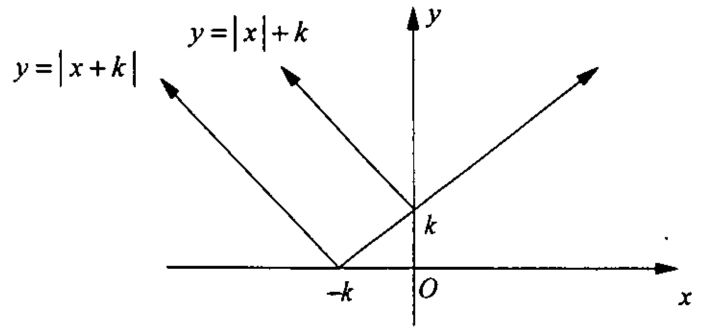
Answers : (a) $2x - y + 3z = 9$ (accept any correct form) *(C2)*

(b) $a = \frac{7}{2}$ *(C2)*

4. $p(27 \le X \le 34) = p\left(\frac{27-30}{2} \le Z \le \frac{34-30}{2}\right)$ *(M1)*
$= p(-1.5 \le Z \le 2)$ *(A1)*
$= 0.433... + 0.477...$ *(M1)*
$= 0.910$ (3 s.f.) (accept 0.911) *(A1)*

Answer : 0.910 *(C4)*

5. $|x+k| = |x| + k$



(M2)

From the graph, $x \geq 0$. *(A2)*

OR $|x+k| = |x| + k$

$\Rightarrow |x+k|^2 = (|x|+k)^2$ *(M1)*

$\Rightarrow x^2 + 2kx + k^2 = x^2 + 2k|x| + k^2$ *(M1)*

$\Rightarrow x = |x|$ *(M1)*

$\Rightarrow x \geq 0$ *(A1)*

Answer : $x \geq 0$ *(C4)*

6. $V = \pi\int_0^k e^{2x}dx$ *(M1)*

$= \frac{\pi}{2}\left[e^{2x}\right]_0^k$ *(A1)*

$= \frac{\pi}{2}\left(e^{2k} - 1\right)$ *(M1)(A1)*

Answer : $\frac{\pi}{2}\left(e^{2k} - 1\right)$ *(C4)*

7. $AH = 5\,\text{cm}, HC = 3\sqrt{5}\,\text{cm}, AC = 2\sqrt{13}\,\text{cm}$ *(A2)*

Note: Award *(A2)* for all 3 correct, *(A1)* for 2 correct.

$\cos A\hat{H}C = \frac{AH^2 + CH^2 - AC^2}{2(AH)(CH)}$ *(M1)*

$= \frac{25 + 45 - 52}{30\sqrt{5}}$

i.e. $A\hat{H}C = 74.4°$(to the nearest one-tenth of a degree) *(A1)*

Answer : 74.4° *(C4)*

8. $\alpha+\beta=k, \quad \alpha\beta=k+1$ *(A1)*

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$k^2-2(k+1)=13$ *(M1)*

$k^2-2k-15=0$

$(k-5)(k+3)=0$

$k=5$ or $k=-3$ *(A2)*

Answers : $k=5$ or $k=-3$ *(C2)(C2)*

9. $\dfrac{3x-4}{x^2-x}=\dfrac{A}{x}+\dfrac{B}{x-1}$ *(M1)*

$A(x-1)+Bx=3x-4$, for all x. *(M1)*

Put $x=0: A=4$ *(A1)*

Put $x=1: B=-1$ *(A1)*

Therefore $\dfrac{3x-4}{x^2-x}=\dfrac{4}{x}-\dfrac{1}{x-1}$

Answer : $\dfrac{4}{x}-\dfrac{1}{x-1}$ *(C4)*

10. $4-9x^2>0$ *(M1)*

$x^2<\dfrac{4}{9}$ *(M1)*

Domain $=\left\{x : -\dfrac{2}{3}<x<\dfrac{2}{3}\right\}$ **OR** $\left\{x : |x|<\dfrac{2}{3}\right\}$ *(A2)*

Answer : $\left\{x : -\dfrac{2}{3}<x<\dfrac{2}{3}\right\}$ **OR** $\left\{x : |x|<\dfrac{2}{3}\right\}$ *(C4)*

11. $\displaystyle\int_1^k\left(1+\frac{1}{x^2}\right)\mathrm{d}x=\frac{3}{2}$

$\left[x-\dfrac{1}{x}\right]_1^k=\dfrac{3}{2}$ *(M1)*

$k-\dfrac{1}{k}=\dfrac{3}{2}$ *(A1)*

$2k^2-3k-2=0$

$(2k+1)(k-2)=0$ *(M1)*

$k=2$ since $k>1$ *(A1)*

Answer : $k=2$ *(C4)*

12. (a) $AB = I$

$(AB)_{11} = 1 \Rightarrow a - 12 + 6 = 1,$ giving $a = 7$ *(A1)*

$(AB)_{22} = 1 \Rightarrow -16 + 5b + 7 = 1,$ giving $b = 2$ *(A1)*

(b) The system is $BX = \begin{pmatrix} 5 \\ 0 \\ 6 \end{pmatrix}$ where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

Then, $X = A\begin{pmatrix} 5 \\ 0 \\ 6 \end{pmatrix} = \begin{pmatrix} 7 & -4 & -6 \\ -8 & 5 & 7 \\ -5 & 3 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ 0 \\ 6 \end{pmatrix}$. *(M1)*

Thus, $x = -1, y = 2, z = -1$. *(A1)*

Answers : (a) $a = 7, b = 2$ *(C1)(C1)*

(b) $x = -1, y = 2, z = -1$ *(C2)*

13. Required probability $= p(A$ plays a higher ranked team and wins$) +$
$p(A$ plays a lower ranked team and wins$)$ *(M1)*

$= \frac{3}{9} \times \frac{2}{5} + \frac{6}{9} \times \frac{3}{4}$ *(M1)(A1)*

$= \frac{19}{30}$ *(A1)*

Answer : $\frac{19}{30}$ *(C4)*

14. $p(x) = (ax + b)^3$

$p(-1) = -1 \Rightarrow (b - a)^3 = -1$ *(M1)*

$\Rightarrow b - a = -1$ *(A1)*

$p(2) = 27 \Rightarrow (2a + b)^3 = 27$

$\Rightarrow 2a + b = 3$ *(A1)*

Thus, $a = \frac{4}{3}, b = \frac{1}{3}$. *(A1)*

Answers : $a = \frac{4}{3}, b = \frac{1}{3}$ *(C4)*

15. $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ *(M1)*

$= \frac{a(\sin\theta)}{a(1-\cos\theta)}$

$= \frac{\sin\theta}{1-\cos\theta}$ *(A1)*

At $\theta = \frac{\pi}{2}, \frac{dy}{dx} = 1, x = a\left(\frac{\pi}{2} - 1\right)$ and $y = a$ *(A1)*

The equation of the normal is $x + y = \frac{\pi a}{2}$. *(A1)*

Answer : $x + y = \frac{\pi a}{2}$ *(C4)*

16. $a(t) = -\frac{1}{20}t + 2$

$v(t) = -\frac{1}{40}t^2 + 2t + c$ *(M1)*

$v = 0$ when $t = 0$, and so $c = 0$

Thus, $v(t) = -\frac{1}{40}t^2 + 2t = -\frac{1}{40}t(t-80)$. *(A1)*

Since $v(t) \geq 0$ for $0 \leq t \leq 80$, the distance travelled $= \int_0^{60} v(t)\,dt$ *(M1)*

$= \left[-\frac{1}{120}t^3 + t^2\right]_0^{60}$

$= 60^2\left(1 - \frac{1}{2}\right)$

$= 1800$ m. *(A1)*

Answer : 1800 m *(C4)*

17. The distance from the centre of the circle to the line equals the radius of the circle. *(M1)*

Thus, $\frac{|5k - 1 + 1|}{\sqrt{k^2 + 1}} = 3$. *(A1)*

Then, $|5k| = 3\sqrt{k^2 + 1}$.

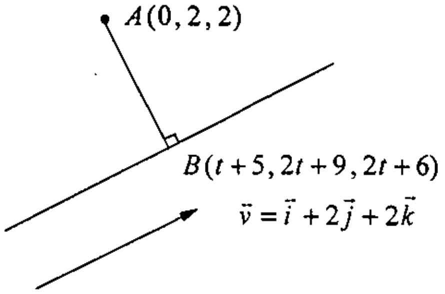
$25k^2 = 9(k^2 + 1)$

$16k^2 = 9$

$k = \pm\frac{3}{4}$. *(A2)*

Answers : $k = \pm\frac{3}{4}$ *(C4)*

18.



$\overrightarrow{AB} \cdot \vec{v} = 0$ *(M1)*

Therefore, $\begin{pmatrix} t+5 \\ 2t+7 \\ 2t+4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 0$

$t+5+4t+14+4t+8=0$, *(A1)*

giving $t=-3$ *(A1)*

Then $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, and the required distance $AB = 3$. *(A1)*

Answer : 3 *(C4)*

19. $\dfrac{dy}{dx} = y\tan x + 1, \quad 0 \le x < \dfrac{\pi}{2}$

$\dfrac{dy}{dx} - y\tan x = 1$

$h(x) = \int -\tan x \, dx = \ln(\cos x)$

The integrating factor is $e^{h(x)} = \cos x$. *(M1)*

Now, $\cos x \dfrac{dy}{dx} - y\sin x = \cos x$

$\dfrac{d}{dx}(y\cos x) = \cos x$

$y\cos x = \int \cos x \, dx = \sin x + c$

$y = \tan x + c\sec x$ *(M1)(A1)*

But, $y = 1$ when $x = 0$ giving $c = 1$.

$y = \tan x + \sec x$ *(A1)*

Answer : $y = \tan x + \sec x$ **OR** $y\cos x = \sin x + 1$ *(C4)*

20. Area under parabola $= 2\int_0^a \left(a^2 - x^2\right) dx$ *(M1)*

$= 2\left[a^2 x - \frac{1}{3}x^3\right]_0^a$ *(A1)*

$= \frac{4}{3}a^3$ *(A1)*

Since $PQ = 2a$, the dimensions of the rectangle are $2a \times \frac{2}{3}a^2$. *(A1)*

Answer : $2a \times \frac{2}{3}a^2$ *(C4)*